Soo Hawk

1. Anna and Bella are celebrating their birthdays together. Five years ago, when Bella turned 6 years old, she received a newborn kitten as a birthday present (0 years old at this time). Today the sum of the ages of the two children and the kitten is 30 years. How many years older than Bella is Anna?

2. Luka is making lemonade to sell at a school fundraiser. His recipe requires 4 times as much water as sugar and twice as much sugar as lemon juice. He uses 3 cups of lemon juice. How many cups of water does he need?

3. The digits 2,0,2, and 3 are placed in the expression below, one digit per box. What is the maximum possible value of the expression?


4. Three positive integers are equally spaced on a number line. The middle number is 15, and the largest number is 4 times the smallest number. What is the smallest of these three numbers?

5. What is the value of $\dfrac{1}{3} \cdot \dfrac{2}{4} \cdot \dfrac{3}{5} \cdots \dfrac{18}{20} \cdot \dfrac{19}{21} \cdot \dfrac{20}{22}$?

6. A donkey has a very long piece of pasta. He takes a number of bites of pasta, each time eating 3 inches of pasta from the middle of one piece. In the end, he

has 10 pieces of pasta whose total length is 17 inches. How long, in inches, was the piece of pasta he started with?

7. Three hexagons of increasing size are shown below. Suppose the dot pattern continues so that each successive hexagon contains one more band of dots. How many dots are in the next hexagon?
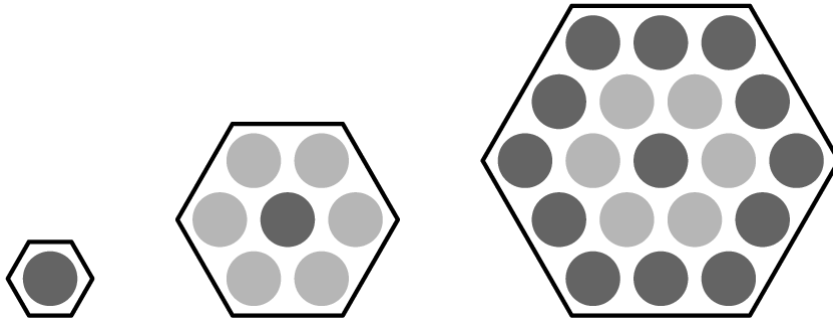
8. What is

$$100 - 98 + 96 - 94 + 92 - 90 + \cdots + 8 - 6 + 4 - 2.$$

Solutions:

1. 5 years ago Bella was 6, and 5 years ago the cat was 0 years old. This means now Bella is 11 years old, and the cat is 5 years old. This means Anna is 30 -11 - 5 = 14 years old since the sum of all their ages is 30. This is 3 years older than Bella, and 3 is the answer.
2. Since he needs twice as much sugar as lemon juice, he needs 2 x 3 = 6 cups of sugar since he needs 3 cups of lemon juice. He needs 4 times as much water as sugar. THis means he needs 4 * 6 = 24 cups of water since he uses 6 cups of sugar.
3. The zero is the key to solving this problem. It must be in the exponent because otherwise the product is 0. Now we can just test out the two possible cases. 2^0 * 3^2 = 9. 2^0 * 2^3 = 8. Since 9 is greater than 8, 9 is the correct answer.
4. Since all the numbers are equally spaced, we can label each of these numbers. The smallest number can be 15 - x, the middle number is 15, and the largest number is 15 + x. This should make sense as then all the numbers will be equally spaced. The largest number is 4 times the smallest number so we can just do algebra to solve.

15 + x = 4 * (15-x)

15 + x = 60 - 4x

5x = 45

x = 9

Now note that x = 9 is not the answer since the problem asks the smallest number. The smallest number is 15 -x which is 15 - 9 = 6. 6 is the answer.

5. The common factors (from 3 to 20 inclusive) of the numerator and the denominator cancel. Write more of the first couple of terms in order to see this better if necessary.

$$\frac{1}{\cancel{3}} \cdot \frac{2}{\cancel{4}} \cdot \frac{\cancel{3}}{\cancel{5}} \cdots \frac{\cancel{18}}{\cancel{20}} \cdot \frac{\cancel{19}}{21} \cdot \frac{\cancel{20}}{22} = \frac{1 \cdot 2}{21 \cdot 22} = \frac{1}{21 \cdot 11} = \boxed{(\text{B}) \ \frac{1}{231}}.$$

6. If he bites from the middle of one piece each time, then he creates an extra piece after each bite. This means he bites the pasta 9 times. The first time he bites there are two pieces, second time he bites there are 3, and so on. He ate 9 * 3 = 27 inches of pasta from these 9 bites. THis means he started with 17 + 27 = 44 inches of pasta.

7. **The best way to solve this problem is to draw the dots around the next hexagon. As a challenge, try to see if you can find a pattern between the number of dots in each hexagon. After drawing the dots on the next hexagon (just draw it on top of the third hexagon) and counting it, we get 37.**
8. **For this problem, we must pair up the numbers so that we can add them all up.**

We can pair

100 - 98 = 2

96 - 94 = 2

92 - 90 = 2

Now the question is how many of these pairs are there? Well how many numbers are there in 100,98,96,94, … 2. There are 50 numbers as we can just divide the sequence by 2 and get 50,49,48, … 1, in which there are 50 numbers. This means there are 25 pairs as 50/2 = 25. 25 pairs of numbers that give a difference of 2. 25 * 2 = 50 which is the answer.